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Stochastic Linear Programming

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Abstract— Operations Research has three major methods called Mathematical Programming Techniques, Stochastic Process Techniques and Statistical Methods. Mathematical Programming plays a vital role among them. This programming has too many branches. Stochastic Programming is one of these branches. Non-linear programming algorithms are classified into two algorithms. They are unconstrained and constrained nonlinear algorithms. In general, there is no algorithm for handling nonlinear models, mainly because of the irregular behaviour of the nonlinear functions. Perhaps the most general result applicable to the problem is the Kuhn Tucker conditions. In constrained non-linear algorithms, stochastic programming techniques solve the non-linear problem by dealing with one or more linear problems that are extracted from the original program. This paper deals with basic concepts in stochastic linear programming. There are two techniques viz. two stage programming and chance constrained programming with an example which is solved in a computer in the Pascal Language.

Index Terms— Khun-Tucker Conditions, Non-linear programming, Mathematical programming, two stage programming, chance Programming, Stochastic process, Pascal language

1 INTRODUCTION

Stochastic or Probabilistic programming deals with situations where some or all of the parameters of the optimization problem are described by stochastic or random or probabilistic variables rather than by deterministic quantities. The sources of random variables may be several, depending on the nature and the type of the problem.

Depending on the nature of equations involved in terms of random variables in the problem a stochastic optimization-problem is called a stochastic linear or dynamic or non-linear programming problem.

The basic idea used in solving any stochastic programming problem is to convert the stochastic problem into an equivalent deterministic problem. The resulting deterministic problem is then solved by using the familiar techniques like linear geometric, dynamic and non-linear programming.

STOCHASTIC LINEAR PROGRAMMING

A stochastic linear programming problem can be stated as follows:

$$\text{Mini } f(x) = C^T X = \sum_{j=1}^{n^2} Cj\ Xj \quad \text{.... (1.1)}$$

subject to

$$A_i^T X = \sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad i = 1,2,....m \quad \text{.... (1.2)}$$

and

$$x_j \geq O \quad j = 1,2,.....n \quad \text{... (1.3)}$$

where c_j, a_{ij} and bi are random variables and x_j are assumed to be deterministic with known probability distributions. Several methods are available for solving the problem stated in (1.1) – (1.3). Here we are dealing with only two methods namely the two stage programming technique and the chance constrained programming technique.

Before dealing we shall see some examples for stochastic programming problems and shall examine the difficulties involved in their solution.

SEQUENTIAL STOCHASTIC PROGRAMMING PROBLEM

Consider the inventory problem in which plans are being made to control the inventory of single item over the time period of n years. It is assumed that the orders to replenish the inventory are placed at the beginning of each year, so that the quantity required comes into inventory before the end of that year. The demand in every year is treated as continuous random variable and also the demands in different years are assumed to be independent variables. If x_j denotes the number of units being ordered, then the cost of procuring x_j units at the beginning of j^{th} year will be,

$$A_j\ \delta_j + c_j\ x_j,$$

$$\text{Where } \delta j = \begin{cases} 0 & \text{for } x_j = 0 \\ 1 & \text{for } x_j > 0 \end{cases} \text{ and}$$

A_j is fixe charge.

There costs associated with the inventory problem are the carrying cost and the stock out cost. Ignoring the accuracy of these we assume that $k_j\ h_j$ and $\pi_j\ s_j$ are the carrying cost and stock-out cost, respectively for the j^{th} year. Here $h_j \geq 0$ denotes the inventory in hand at the end of j^{th} year and $s_j \geq 0$ denotes

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the number of back orders at the end of j^{th} year. Let us define,

Y_1 = Inventory in hand at the beginning of 1^{st} year

v_i = Demand in the i^{th} year

x_i = Quantity order in the i^{th} year

Using these notations, the inventory in hand at the end of j^{th} year is given by

$$h_j = Y_1 + \sum_{i=1}^{j} x_i \sum_{i=1}^{j} v_i \qquad \text{if } h_j \geq 0$$

The number of backorder at the end of j_{th} year will be given by

$$S_j = \sum_{i=1}^{j} v_i - \sum_{i=1}^{j} x_i - y_{1i} \qquad \text{if } S_j > 0$$

It is to be noted that one of the carrying cost and stockout cost disappear in the presence of the other. So let us define a new function,

$$F_j\left(y_1 + \sum_{i=1}^{j} x_i + \sum_{i=1}^{j} v_i\right) = \begin{bmatrix} k_j h_j, & \text{for } h_j \geq 0 \\ \pi_j s_j, & \text{for } s_j > 0 \end{bmatrix}$$

Then the cost of operating the inventory system for the n years time period will be

$$\sum_{j=1}^{n} a_j \left[A_j \delta_j + c_j x_j\right] + \sum_{j=1}^{n} a_j F_j\left(y_1 + \sum_{i=1}^{j} x_i - \sum_{i=1}^{j} v_i\right)$$

Where α_i is the discounting factor. If $\alpha_i = 1$, then there will be no discounting factor. The probability density function for getting a specific set of v_j is

$$\underset{j=1}{\overset{n}{\pi}} \theta(v_j) = \theta(v_1)\theta(v_2)....\theta(v_n)$$

Thus for any given set of $x_j > 0$, the expected cost over n years time period is given by,

$$Z = \int_o^{\infty} \int_o^{\infty} \bullet\bullet \int_o^{\infty} \left[\underset{j=1}{\overset{n}{\pi}} \theta(v_j)\right]\left[\sum_{j=1}^{n} \alpha_j \left[A_j \delta_j + c_j x_j\right] + \sum_{j=1}^{n} \delta_j F_j\left(y_1 + \sum_{i=1}^{j} x_i - \sum_{i=1}^{j} v_i\right)\right]$$

$dv_1...dv_n$

When the decision is made on how much to order for period K, the optimal value of x_k will depend upon y_k. Using the technique of dynamic programming we find out the functions $x^k(y_k)$, where the value of y_k, is given by

$$y_k = y_1 + \sum_{i=1}^{k-1} x_i - \sum_{i=1}^{k-1} v_i$$

TWO STAGE PROGRAMMING TECHNIQUE

INTRODUCTION

The two-stage programming technique converts a stochastic linear programming problem into an equivalent deterministic problem. Assume that the elements b_i are probabilistic. This means that the variable b_i is not known but its probability distribution function with a finite mean b_i is known to us. In this case it is impossible to find a vector X in such a way that $A_i^T X$ will be greater than or equal to b_i (i = 1,2,.....m) for whatever value bi takes. The difference between $A_i^T X$ and b_i a random variable, whose probability distribution function depends on the value of X chosen.

One can now think of associating a penalty for violation we might get for the constraints. In this case, we can think of minimizing the sum of $C^T X$ and the expected value of the penalty. There are several choices for the penalty. One choice is to assume a constant penalty cost of p_i for violating the i^{th} constraint by one unit. Thus the total penalty is given by the expected (mean) value of the sum of the individual penalties, $\sum_{i=1}^{m} E(p_i y_i)$ where E is the expectation and y_i is defined as,

$$y_i = b_i - A_i^T X, \; y_i \geq 0 \qquad I = 1,2,\ldots\ldots m \qquad \ldots (2.1)$$

After adding the mean total penalty cost to the original objective function, the new optimization problem becomes

$$\text{Minimize } C^T X + E(P^T Y) \qquad \ldots (2.2)$$

subject to

$$AX + BY = b \qquad \ldots (2.3)$$

and

$$X \geq 0, \; Y \geq 0 \qquad \ldots (2.4)$$

Where

$$P = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_m \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{bmatrix}$$

and B = I = Identity matrix of order m.

The penalty term in Eqn. (2.2) will be a deterministic quantity in terms of the expected values of y_i, $\bar{y}_i$. For eg. if b_i follows uniform (rectangular) distribution in the range $\left[\bar{b}_i - m_i, \bar{b}_i + m_i\right]$ and $\bar{y}_i$ denotes $\bar{b}_i$ - $A_i^T X$, then the mean penalty cost can be shown to be equal to

$$E(p_i y_i) = p_{i1} + p_{i2} + p_{i3} \qquad ... (2.5)$$

Where

$$p_{i1} = 0 \qquad \text{if } \bar{y}_i \geq m_i \qquad ... (2.6)$$

$$P_{i2} = 0 \int_{s=0}^{(m_i - \bar{y}_i)} \frac{p_i}{2m_i} \, sds \qquad \text{if } -m_i < \bar{y}_i < m_i \qquad \ldots (2.7)$$

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$$P_{i3} = 0 \int_{s=(-m_i-\bar{y}_i)}^{(m_i-\bar{y}_i)} \frac{p_i}{2m_i} sds \quad \text{if } \bar{y}_i < m_i$$

… (2.8)

From (2.7)

$$P_{i2} = \frac{p_i}{2m_i} \frac{s^2}{2} \Big|_{s=0}^{m_i-\bar{y}_i}$$

$$= \frac{p_i}{4m_i}\left(m_i - \bar{y}_i\right)^2$$

… (2.9)

From (2.8)

$$P_{i3} = \frac{p_i}{2m_i} \frac{s^2}{2} \Big|_{s=(-m_i-\bar{y}_i)}^{(m_i-\bar{y}_i)}$$

$$= \frac{pi}{4mi}\left[\left(m_i - \bar{y}_i\right)^2 - \left(-m_i - \bar{y}_i\right)^2\right]$$

$$= \frac{pi}{4mi}\left[\left(m_i - \bar{y}_i - m_i - \bar{y}_i\right)\left(m_i - \bar{y}_i + m_i + \bar{y}_i\right)\right]$$

$$= \frac{pi}{4mi}\left(-4\bar{y}_i m_i\right)$$

$$= -p_i \bar{y}_i$$

… (2.10)

Using (2.6), (2.6) and (2.10) in (2.5) we have,

$$E(p_i y_i) = \frac{pi}{4mi}\left(m_i - \bar{y}_i\right)^2 - p_i \bar{y}_i$$

… (2.11)

Which is a quadratic function of deterministic variable $\bar{y}_i$.

To convert the problem stated in equations (2.1) to (2.4) to a fully deterministic one, the probability constraints eqn. (2.3) have to be written either in a deterministic form like $\bar{y}_i = \bar{b}_i - A_i^T X$, or interpreted as a two-stage problem as follows.

FIRST STAGE

First estimate the vector b, and find the vector X by solving the problem stated in eqns. (1.1) to (1.3).

SECOND STAGE

Then observe the value of b and hence its discrepancy from the previous guess vector and find the vector y = Y (b,X) by solving the second stage problem:

Find Y which minimize $P^T Y$

Subject to

$y_i = b_i - A_i^T X$, I = 1,2,….m

… (2.12)

and $y_i \geq 0$, I = 1,2….m

where b_i and X are known now.

Thus the two-stage formulation can be interpreted to mean that a non-negative vector X must be found before the actual values of b_i (i = 1,2…..m) are known, and that when they are known, a recourse Y must be found by solving the second stage problem of Equations (2.12). Hence a general two stage problem can be stated as follows:

Minimize $C^T X + E\left[\min\left(P^T Y\right)\right]$

subject to

… (2.13)

$$\underset{m \times n_1}{A}\ \underset{n_1 \times 1}{X} + \underset{m \times n_2}{B}\ \underset{n_2 \times 1}{Y} \geq \underset{m \times 1}{b}$$

Where b is a random m-dimensional vector with known probability distribution F(b) and probability density function dF(b) = f(b)

The following assumptions are generally made to solve this problem.

(i) The penalty cost vector P is a known deterministic vector and

(ii) There exists a nonempty convex set S consisting of

non-negative solution vectors X such that for each b there exists a solution vector Y(b) so that the pair [X, Y(b)] is feasible. The 2nd assumption is called the

Assumption of permanent feasibility. By defining,

$$\overset{D}{mx\left(n_1 + n_2\right)} = [A, B]$$

… (2.14)

$$\overset{Q}{\left(n_1 + n\right)x1} = \begin{bmatrix} C \\ P \end{bmatrix}$$

… (2.15)

and

$$\overset{Z(b)}{\left(n_1 + n_2\right)x1} = \overset{X}{\left[Y(b)\right]}$$

… (2.16)

The two stage problem stated in equation (2.13) can be as follows:

Minimize $\int Q^T Z(b) f(b)$ = expected cost

Subject to D z(b) $\geq$ b

And z(b) $\geq$ 0 V b

<u>Example:</u> Find the optimal values of factory production (x_1) excess supply (x_2) and the amount purchased from outside (x_3) of commodity, for which the market demand (r) is a uniformly distributed r.v. with a density function of f(r) = $\frac{1}{(80-70)}$.

Each unit produced in the factory costs Rs. 1 whereas each unit purchased from outside costs Rs.2. The constraints are that

(i) The total supply of the commodity ($x_1 + x_3$) should not be less than the demand (r) and

(ii) Due to storage space and other restrictions, the amount of production in the factory (x_1) plus the amount stored (x_4) should be equal to 100 units.

Solution:

This problem can be stated as follows:

Minimize $f = x_1 + 2.x_3$

= cost of production + cost of purchasing out-side.

Subject to

$x_1 + x_4 = 100$

and $x_1 + x_3 - x_2 = r$

where $x_i > 0$, i = 1,2,3,4.

$$f(r) = \frac{1}{(80-70)} = \frac{1}{10}$$

since the second constraint is probabilistic, we assume permanent feasibility condition for it. This means that it is possible to choose x_3, outside purchase and x_2, excess supply for whatever may be the feasible values of x_1, factory production and x_4, amount stored.

It can be seen that if $x_1 > r$ for any particular value of r, then $x_3 = 0$ gives the minimum value of f. However, if $x_1 \le r$ then $x_3 = r - x_1$ gives the minimum values of f since x1 is cheaper than x3. Thus we obtain

$$\underset{(x_3)}{\min imum}\ f = \begin{bmatrix} x_1 & \text{if } x_1 > r \\ x_1 + 2(r - x_1) & \text{if } x_1 \le r \end{bmatrix}$$

Since the market demand is probabilistic, we have to consider the following three cases.

Case (i): When $x_1 > 80$ i.e., when $x_1 > r$

E (minimum f) = E(x_1) = x_1

Case (ii): When $x_1 < 70$ i.e., whe $x_1 < r$

E (minimum f) = E[x_1 + 2 (r-x_1)]

$$= \int_{70}^{80} (x_1 + 2r - 2x_1) f(r)\,dr = \int_{7}^{80} \left(\frac{2r - x_1}{10}\right) dr$$

$$= \frac{r^2 - r x_1}{10}\Bigg|_{70}^{80} = \frac{6400 - 80x_1 - 4900 + 70x_1}{10}$$

$$= \frac{1500 - 10x_1}{10} = 150 - x_1$$

Case (iii): When $70 \le x_1 \le 80$. Here the demand may be less than, equal to or greater than x_1:

E(minimum f) =

$$\int_{70}^{x_1} x_1 f(r)\,dr + \int_{x_1}^{80} [x_1 + 2(r - x_1)] f(r)\,dr$$

$$= \frac{x_1 r}{10}\Bigg|_{70}^{x_1} + \frac{1}{10}\left[x_1 r + r^2 - 2x_1 r\right]_{x_1}^{80}$$

$$= \frac{x_1^2}{10} - \frac{70\,x_1}{10} + \frac{1}{10}\left[r^2 - x_1 r\right]_{x_1}^{80}$$

$$= \frac{x_1^2 - 70x_1}{10} + \frac{1}{10}\left[6400 - 80x1 - x_1^2 + x_1^2\right]$$

$$= \frac{x_1^2 - 70x_1}{10} + \frac{1}{10}\left[6400 - 80x_1\right]$$

$$= \frac{1}{10}\left[x_1^2 - 70x_1 + 6400 - 80x_1\right]$$

$$= \frac{1}{10}\left[x_1^2 - 150x_1 + 6400\right]$$

$$= \frac{1}{10}[75 - x_1]^2 + 77.5$$

Hence the total expected cost function is a quadratic function in x_1.

Its minimum is given by,

$$E(\min f) = \frac{1}{10}\left[x_1^2 - 150x_1 + 6400\right]$$

$$\frac{dE}{dx_1} = \frac{1}{10}[2x_1 - 150]$$

$$= \frac{x_1}{5} - 15$$

$$\frac{dE}{dx_1} = 0 = \frac{x_1}{5} - 15 = 0$$

$$\Rightarrow \frac{x_1}{5} = 15$$

$$\Rightarrow x_1 = 75$$

Hence this value satisfies the first constraint also we obtain the optimum solution as

$x_1 = E(r) = 75$, $x_2 = 0$

$x_3 = r - 75$ with $E(x_3) = 0$,

$x_4 = 25$.

CHANCE – CONSTRAINED PROGRAMMING TECH-

NIQUE

An important class of stochastic programming problems is the chance-constrained problems. These problems were initially studied by A. Charnes and W.W. Cooper. In a stochastic programming problem some constraints may be deterministic and the remaining may involve random elements. On the other hand, in a chance-constrained programming problem, the latter set of constraints is not required to always hold, but these must hold simultaneously or individually with given probabilities. In other words, we are given a set of probability measure indicating the extent of violation of the random constraints. The general chance-constrained linear program is of the form:

$$\text{Minimize } f(x) = \sum_{j=1}^{n} c_j x_j \quad \text{... (3.1)}$$

Subject to

$$P\left[\sum_{j=1}^{n} ai_j\ x_j \le b_i\right] \ge b_i\Big] \ge p_i, \quad i = 1,2,....m \quad \text{... (3.2)}$$

And $x_j \geq 0$, j = 1,2.....m ... (3.3)

Where c_j, a_{ij}, b_i are random variables and p_i are specified probabilities. Eqn. (3.2) indicate that the i[th] constraint

$$\sum_{j=1}^{n} ai_j x_j \le b_i$$

has to be satisfied with a probability of atleast p_i where $0 \leq p_i \leq 1$. Assume that the decision variables x_j are deterministic. First we consider the special case where only c_j or a_{ij} or b_j are random variables. After we consider the case in which c_j, a_{ij} and bi are all random variables. Further we shall assume that all the random variables are normally distributed with known mean and standard deviations.

ONLY a_{ij}'s ARE RANDOM VARIABLES

Let $\bar{a}_{ij}$ and Var(a_{ij}) = $\sigma^2_{a_{ij}}$ be the mean and the variance of the normally distributed random variables a_{ij}. Also assume that the multivariate distribution of a_{ij}, i = 1,2...m, j = 1,2...n is also known along with the covariance, Cov (a_{ij}, a_{kl}) between the random variables a_{ij} and a_{kl}. Define quantities d_i as

$$d_i = \sum_{j=1}^{n} ai_j\ x_j \qquad i = 1,2....m \quad \text{... (3.4)}$$

Since a_{i1}, a_{i2},...... ain are normally distributed, and x_1, x_2 .... x_n are constants d_i will also be normally distributed with the mean value of

$$\bar{d}_i = \sum_{j=1}^{n} \bar{a}_{ij}\ x_j \qquad i = 1,2,...m \quad \text{... (3.5)}$$

And a variance of

$$\text{Var}(d_i) = \sigma^2_{d_i} = X^T v_i X \quad \text{... (3.6)}$$

Where V_i is the i[th] covariance matrix defined as

$$\begin{pmatrix} \text{Var}(a_{i1}) & (\text{Cov}(a_{i1}, a_{i2}) & \ldots & \text{Cov}(a_{i1}, a_{in}) \\ (\text{Cov}(a_{i1}, a_{i2}) & \text{Var}(a_{i2}) & \ldots & \text{Cov}(a_{i2}, a_{in}) \\ \ldots & \ldots & \ldots & \ldots \\ (\text{Cov}(a_{in}, a_{i1}) & \text{Cov}(a_{in}, a_{i2}) & \ldots & \text{Var}(a_{i2}) \end{pmatrix} \quad \text{... (3.7)}$$

The constraint or equation (3.2) can be expressed as

$$P[d_i \le b_i] \ge p_i$$

i.e.

$$P\left[\frac{d_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}} \le \frac{b_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}}\right] \ge p_i \qquad i = 1,2....m \quad \text{...(3.8)}$$

where $\frac{d_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}}$ can be a standard normal variate with a mean of zero and a variance of one. Thus the probability of d_i smaller than or equal to b_i can be

$$P[d_i \le p_i] = \phi\left(\frac{b_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}}\right) \quad \text{... (3.9)}$$

Where $\phi(x)$ represents the cumulative distribution function of the standard normal distribution evaluated at x. If e_i denotes the value of the standard normal variable at which

$$\Phi(e_i) = p_i \quad \text{... (3.10)}$$

Then the constraints in Eqn. (3.8) can be stated as

$$\phi\left(\frac{b_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}}\right) \ge \theta(e_i) \qquad i = 1,2,....m \quad \text{... (3.11)}$$

These inequalities will be satisfied only if,

$$\left(\frac{b_i - \bar{d}_i}{\sqrt{\text{var}(d_i)}}\right) \ge e_i$$

(or)

$$\bar{d}_i + e_i\sqrt{\text{var}(d_i)} - b_i \le 0, \qquad i = 1,2,...m \quad \text{... (3.12)}$$

By substituting Eqn. (3.5) and 3.6) in 3.12)

$$\sum_{j=1}^{n} \bar{a}_{ij}\ x_j + e_i \sqrt{X^T V_i X} - b_i \le 0 \qquad i = 1,2,.....m$$

... (3.13)

These are the deterministic non-linear constraints equivalent to the original stochastic linear constraints.

Thus the solution of the stochastic programming problem stated in Eqns. (3.1) to (3.3) can be obtained by solving the equivalent deterministic programming problem.

Minimize $f(x) = \sum_{j=1}^{n} c_j\ x_j$ subject to

$$\sum_{j=1}^{n} \bar{a}_{ij}\ x_j + e_i \sqrt{X^T V_i X} - b_i \le 0, \qquad i = 1,2,....m \quad ... (3.14)$$

and $x_j > 0$, j = 1,2,.....n.

If the normally distributed random variables a_{ij} are independent the covariance terms will be zero and equation (3.7) reduces to a diagonal matrix as

$$\begin{bmatrix} Var(a_{i1}) & 0 & 0 \\ 0 & Var(a_{i2}) & 0 \\ 0 & 0 & Var(a_{i3}) \end{bmatrix}$$

... (3.15)

In this case the constraints of Eqn. (3.13) reduce to

$$\sum_{j=1}^{n} \bar{a}_{ij}\ x_j + e_i \sqrt{\sum_{j=1}^{n} \left[Var(ai_j)\, x_j^2\right]} - b_i < 0$$

i = 1,2,....m ... (3.16)

ONLY b_i's ARE RANDOM VARIABLES:

Let $\bar{b}_i$ and $var(b_i)$ denote the mean and variance of the normally distributed random variable b_i. The constraints of equation (3.2) can be restated as

$$P\left[\sum_{j=1}^{n} \bar{a}_{ij}\ x_j \le b_i\right] = P\left[\frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}} \le \frac{b_i - \bar{b}_i}{\sqrt{var(b_i)}}\right]$$

$$= P\left[\frac{b_i - \bar{b}_i}{\sqrt{var(b_i)}} \ge \frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}}\right] \ge p_i \qquad i = 1,2,...m \quad ... (3.17)$$

Where $\left[\frac{(b_i - \bar{b}_i)}{\sqrt{var(b_i)}}\right]$ is a standard normal variable with zero mean and unit variance. The inequalities (3.17) can also be stated as,

$$1 - P\left[\frac{b_i - \bar{b}_i}{\sqrt{var(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}}\right] \ge p_i \qquad i = 1,2,....m$$

(or)

$$P\left[\frac{b_i - \bar{b}_i}{\sqrt{var(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}}\right] \le 1 - p_i \qquad i = 1,2,...m \quad ... (3.18)$$

If E_i represents the value of the standard normal variate at which $\Phi(E_i) = 1 - p_i$.

The constraints in Eqn. (3.18) can be expressed as

$$\Phi\left(\frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}}\right) \le \Phi(E_i) \qquad i = 1,2,.....m$$

... (3.19)

These inequalities will be satisfied only if

$$\frac{\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i}{\sqrt{var(b_i)}} \le E_i \qquad i = 1,2,...m$$

(or)

$$\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i - E_i \sqrt{var(b_i)} \le 0, \qquad i = 1,2,.....m$$

... (3.20)

Thus the stochastic linear programming problem stated in equations (3.1) to (3.3) is equivalent to the following deterministic linear programming problem.

Minimize $f(x) = \sum_{j=1}^{n} c_j\ x_j$

Subject to $\sum_{j=1}^{n} a_{ij}\ x_j - \bar{b}_i - E_i \sqrt{var(b_i)} \le 0, \qquad i = 1,2,...m$

and

$x_j > 0, \quad j = 1,2,...m$

... (3.21)

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CONCLUSION

Stochastic programming techniques are useful whenever the parameters of the optimization problem are stochastic or random in nature. The basic idea used in all the stochastic optimization techniques is to convert the problem into an equivalent deterministic problem so that the techniques of linear can be applied to find the optimum solution. In stochastic programming problems, the two stage programming and the chance constrained programming techniques are presented for solving a stochastic linear programming problem. On the other hand the solution of stochastic nonlinear programming problems is considered using chance constrained programming technique only.

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